Reflection Traveltime Inversions in VTI Media

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Abstract

Anisotropy parameters in a VTI medium can be obtained by normal-moveout velocity analysis performed on short-spread or long-spread reflection-seismic data, in combination with check-shot or well-log data. Analysis of three traveltime approximations to the actual reflection traveltime shows that each traveltime approximation has its own requirements for spread length and subsurface anisotropic parameters. When $\varepsilon - \delta = 0$ (elliptical anisotropy), all three approximations are the same as the actual reflection traveltime. When $|\ \varepsilon - \delta\ | < 0.2$, the hyperbolic reflection traveltime approximates well to the exact traveltime on short-spread seismic data, the modified three-term Taylor series approximates well on intermediate-spread data, and the nonhyperbolic traveltime approximates well on long-spread seismic data. When $|\ \varepsilon - \delta\ | > 0.2$, the hyperbolic and modified three-term Taylor-series approximate traveltimes largely deviate from the actual reflection traveltime. The results of the three reflection traveltime inversions by semblance scan demonstrate that the estimated anisotropy parameters are always less than but close to the actual anisotropic parameters, and that the hyperbolic traveltime and modified three-term Taylor-series inversion are not suitable for $|\ \varepsilon - \delta\ | > 0.2$.

Introduction

There are various reflection-traveltime inversion approaches for estimating anisotropy parameters but each has its own assumptions and limitations. It is necessary to understand these assumptions and limitations in order to guide their application. Thomsen (1986) has derived relations between normal-moveout (NMO) velocities and anisotropy parameters in a homogeneous anisotropic layer. We can use these approximations to obtain anisotropy parameters in a VTI medium by NMO-velocity analysis performed on short-spread or long-spread seismic data in combination with check-shot or well-log data. Besides hyperbolic NMO-velocity analysis, a popular approach for estimating anisotropy using reflection seismic data is a three-term Taylor series approximation to the reflection moveout curve.

It has been shown that it may not be necessary to know the individual values of the anisotropic parameters for P-wave time processing (Alkhalifah and Tsvankin, 1995). All time-processing steps, including NMO, DMO, and time migration, are fully determined by two parameters (NMO velocity, V_{NMO}, and η, or V_{NMO} and horizontal velocity, V_{hor}) that describe the reflection moveout. Based on the nonhyperbolic moveout equation developed by Tsvankin and Thomsen (1994), a 2-D semblance scan can be used to estimate anisotropy parameters. For convenience, we refer to this method as nonhyperbolic reflection-traveltime inversion.

In this paper, we compare the traveltime approximations of three reflection-traveltime inversion methods (hyperbolic traveltime inversion, three-term Taylor-series inversion and nonhyperbolic inversion) with the exact traveltimes in VTI media. We then carry out these three inversions on a synthetic seismic data example and compare the estimated anisotropy parameters with the true anisotropy parameters. Finally, we formulate some conclusions for guiding the application of these approximations.

Reflection Traveltime Approximations

An approximation of the exact eikonal equation in the quasi-compressional case for so-called weak anisotropy (Daley, 2001) may be written as

$$G_{qp}(p,q,x,z) \approx A_{11}p^2 + A_{33}q^2 + \frac{A_D p^2 q^2}{(A_{11}-A_{55})p^2 + (A_{33}-A_{55})q^2} \approx 1 \tag{1}$$

$$A_D = (A_{13}+A_{55})^2 - (A_{11}-A_{55})(A_{33}-A_{55}) \tag{2}$$

$$p = \frac{\sin\theta}{V_n(\theta)} \text{ and } q = \frac{\cos\theta}{V_n(\theta)}. \tag{3}$$

where $A_{33}=\alpha_0^2$, $A_{55}=\beta_0^2$, $A_{11}=(1+2\varepsilon)\alpha_0^2$, $A_{13}=\sqrt{(\alpha_0^2-\beta_0^2)^2+2\delta\alpha_0^2(\alpha_0^2-\beta_0^2)}-\beta_0^2$, α_0 and β_0 are vertical velocities for P- and S-waves, δ and ε are Thomson's anisotropy parameters. Phase velocity, $V_n(\theta)$, group angle, φ, and group velocity, V_{ray}, are given by Thomsen (1986) as:

$$\tan\varphi=\left(\tan\theta+\frac{1}{V_n(\theta)}\frac{dV_n(\theta)}{d\theta}\right)\Big/\left(1-\frac{\tan\theta}{V_n(\theta)}\frac{dV_n(\theta)}{d\theta}\right) \tag{5}$$

$$V_{ray}^2(\varphi)=V_n^2(\theta)+\left(\frac{dV_n(\theta)}{d\theta}\right)^2. \tag{6}$$

According to equations (1) to (6), we develop a multilayer ray-tracing code for modelling real traveltime-offset curves (solid line shown in Figure 1).

The hyperbolic reflection-traveltime approximation is given by:

$$t^2(x)=t_0^2+\frac{x^2}{V_{NMO}^2} \tag{7}$$

where

$$V_{NMO}^2(P)=\alpha_0^2(1+2\delta). \tag{8}$$

V_{NMO} is NMO velocity for P- and S-waves; t_0 and t are the two-way traveltimes for zero-offset and offset x.

Tsvankin and Thomsen (1994) derive a three-term Taylor-series long-spread approximation for reflection moveouts of the P- and SV-waves in a single layer in the limit of weak anisotropy:

$$t^2(x)=t_0^2+A_2x^2+\frac{A_4x^4}{1+\left(\frac{x}{v_0t_0}\right)^2} \tag{9}$$

where v_0 is vertical velocity for P- or SV-waves and the parameters A_2 and A_4 are Taylor-series coefficients.

For the P-wave,

$$A_2=\frac{1}{V_{NMO}^2}=\frac{1}{\alpha_0^2(1+2\delta)} \qquad A_4(P)=-\frac{2(\varepsilon-\delta)}{t_{P0}^2\alpha_0^4} \tag{10}$$

If one ignores the contribution of the vertical shear-wave velocity, which is negligible (Tsvankin and Thomsen, 1994; Alkhalifah and Larner, 1994; Tsvankin, 1995), we have the quadric moveout coefficient for P-waves in a single VTI layer:

$$t^2(x)=t_0^2+\frac{x^2}{V_{NMO}^2}-\frac{2\eta x^4}{V_{NMO}^2[t_0^2V_{NMO}^2+(1+2\eta)x^2]}=t_0^2+\frac{x^2}{V_{NMO}^2}-\frac{[V_{hor}^2-V_{NMO}^2]x^4}{V_{NMO}^2[t_0^2V_{NMO}^4+V_{hor}^2x^2]} \tag{11}$$

where

$$V_{NMO}^2(P)=\alpha_0^2(1+2\delta) \qquad \eta=0.5\left(\frac{V_h^2}{V_{NMO}^2}-1\right)=\frac{\varepsilon-\delta}{1+2\delta} \tag{12}$$

Figure 1 shows the reflection-traveltime approximations of equations (7) (dotted line), (9) (dashdot line) and (11) (dashed line) to the exact traveltime (solid line) in the limit of weak anisotropy. When $\varepsilon-\delta=0$, the three approximations are the same as the exact traveltime [Figure1(c) and (f)]. When $0<\varepsilon-\delta\leq 2.0$, these three closely approximate the exact traveltime for the short spread [Figure1 (b)], but deviation of equation (7) and equation (9) from the actual traveltime increases with spread length [Figure1 (e)]. When $\varepsilon-\delta>2.0$, only equation (11) approximates the actual reflection traveltime while the traveltimes from equation (7) and (9) largely deviate from the actual reflection traveltime, even for a short spread. This demonstrates that the hyperbolic reflection traveltime and modified three-term Taylor-series reflection traveltime are not suitable for estimating anisotropic parameters when $\varepsilon-\delta>0.2$.

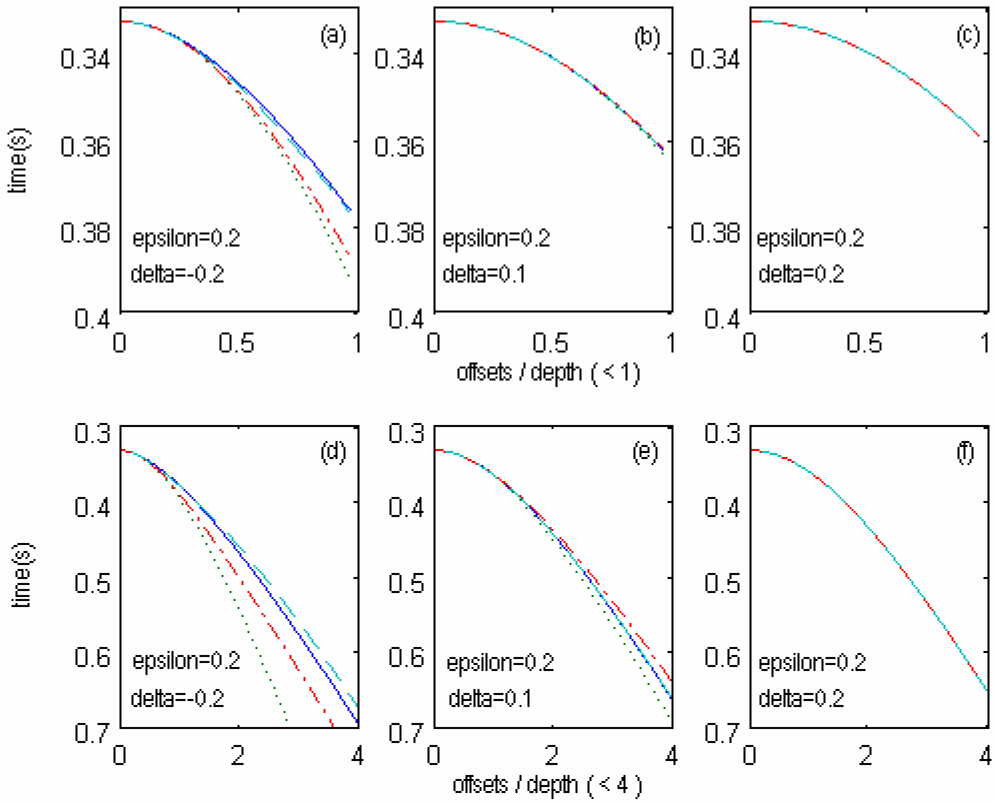

Figure 1. Reflection-traveltime approximation to the true reflection traveltime over a short spread (upper panel) and a long spread (lower panel). Solid line: the exact traveltime; dotted line: hyperbolic traveltime approximation; dash-dot line: three-term Taylor-series approximation; dashed line: nonhyperbolic approximation.

Reflection Traveltime Inversions

The input CMP gather for anisotropy-parameter inversion contains a single reflection from a flat interface. The depth of this interface is 500 m. Vertical P- and S-wave velocities above the reflector are 3000 m/s and 1500 m/s, respectively. The values of the anisotropy parameter δ range from –0.2 to 0.2 at an increment of 0.02. The value of the anisotropy parameter ε is fixed at 0.2. Semblance scanning is employed to pick up normal moveout velocity. For all three inversions, the scanning increments are: for normal moveout velocity, V_{NMO}, 5 m; for zero-offset two-way traveltime, t_0, 0.001 ms, and for horizontal velocity, V_{hor}, 5 m.

Figure 2 shows the crossplots of (a) estimated δ versus true δ and (b) estimated ε versus true δ. For these three reflection traveltime inversions, the error of the estimated anisotropic parameter δ is given by:

$$\Delta\delta = \frac{\sqrt{1+2\delta}}{\alpha_0}\Delta V_{NMO} \tag{13}$$

It is shown in Figure 2(a) and equation (13) that the error in the estimate of the anisotropy parameter δ mainly depends on the accuracy of the picked NMO velocity and is multiplied by $\sqrt{1+2\delta}/\alpha_0$ when $\Delta V_{NMO} \neq 0.0$. Note that the large deviation of the dotted line (hyperbolic) and the dash-dot line (three-term) from the exact parameter, δ, when $\delta < 0$ (i.e. $\varepsilon - \delta > 0.2$) is caused by the deviation of the approximate from from the exact reflection traveltime.

The error in the estimate of the anisotropy parameter ε for the three-term Taylor-series inversion is given by:

$$\Delta\varepsilon = \left(\frac{1+4\varepsilon-6\delta}{\sqrt{1+2\delta}}\right)\frac{1}{\alpha_0}\Delta V_{NMO} - \frac{1}{2(1+2\delta)^2}\Delta A \tag{14}$$

where $A = -2(\varepsilon-\delta)(1+2\delta)^2$. The dash-dot line in Figure 2(b) shows that the error in the estimate of the anisotropic parameter ε is more complicated than for the dashed line and mainly depends on the combined accuracy of picked V_{NMO} and A.

The error in the estimate of anisotropy parameter ε for the nonhyperbolic inversion is given by:

$$\Delta\varepsilon = \frac{\sqrt{1+2\varepsilon}}{\alpha_0}\Delta V_{hor}. \tag{15}$$

The error in the estimated anisotropic parameter ε [dashed line in Figure 2(b)] mainly depends on the accuracy of picked horizontal velocity and is multiplied by $\sqrt{1+2\varepsilon}/\alpha_0$ when $\Delta V_{hor} \neq 0.0$.

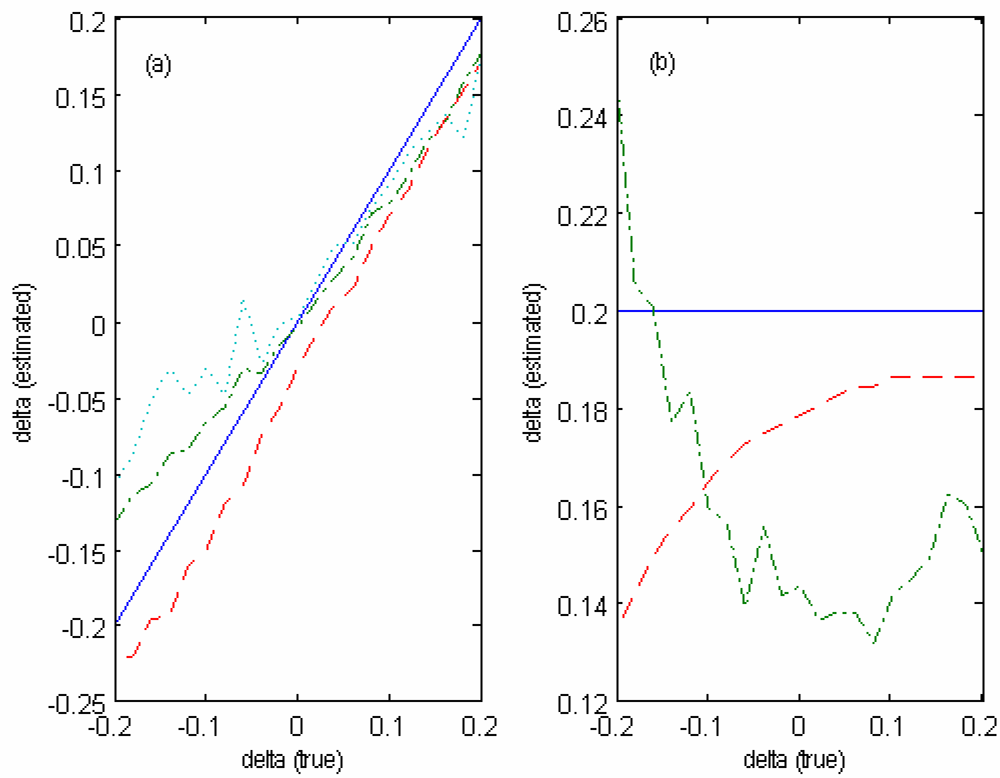

Figure 2. Reflection-traveltime inversion for anisotropic parameters. The crossplots of (a) estimated δ vs true δ; and (b) estimated ε vs true δ. Solid line: the exact anisotropic parameters; dotted line: hyperbolic traveltime inversion; dash-dot line: three-term Taylor-series inversion; dashed line: nonhyperbolic inversion.

Conclusions

The accuracy of reflection-traveltime inversion by semblance scan depends on semblance measures and the choice of inversion approaches. Each traveltime approximation has its own requirements for spread length and subsurface anisotropic parameters. The results of the three traveltime inversions by semblance scan for this seismic example demonstrate that the values of estimated anisotropy parameters always are less than but close to those of the true anisotropy parameters, and that the hyperbolic traveltime inversion and modified three-term Taylor-series inversion are not suitable for estimating anisotropy parameters when $|\varepsilon - \delta| > 0.2$.

Acknowledgments

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